$$(1+t^2)\,y'' + 2t\,y - 2y = 0$$ 1

$$y = \sum_{k=0}^{\infty} a_k t^k$$

$$y' = \sum_{k=\cancel{0}}^{\infty} k\,a_k t^{k-1}$$

$$y'' = \sum_{k=2}^{\infty} k(k-1)\,a_k t^{k-2}$$

$$\sum_{k=2}^{\infty} k(k-1)\,a_k\, t^{k-2} + \sum_{k=2}^{\infty} k(k-1)\,a_k t^k + \sum_{k=1}^{\infty} 2k\,a_k t^k - \sum_{k=0}^{\infty} 2\,a_k t$$

$$\sum_{k=0}^{\infty} (k+2)(k+1)\,a_{k+2}\, t^k + \sum_{k=2}^{\infty} k\,(k-1)\,a_k t^k + \sum_{k=1}^{\infty} 2k\,a_k t^k - \sum_{k=0}^{\infty} 2a_k$$

$$(2a_2 - 2a_0) + \left(6a_3 + \cancel{2a_1} - \cancel{2a_1}\right)t + \sum_{k=2}^{\infty}\Big[(k+2)(k+1)\,a_{k+2} + k(k-1)\,a_k + 2k\,a_k - 2\,a_k\Big]t$$

$\Rightarrow$

$a_0 = a_0$

$a_1 = a_1$

$a_2 = a_0$

$\boxed{a_3 = 0} \Rightarrow$ todos los impares son 0.

$$(k+2)(k+1)\,a_{k+2} = \left(2 - 2k - k^2 + k\right) a_k$$

$$a_{k+2} = \frac{-\left(k^2 + k - 2\right)}{(k+2)\,(k+1)}\,a_k$$

$$a_{k+2} = -\frac{(k-1)\,\cancel{(k+2)}}{\cancel{(k+2)}\,(k+1)}\,a_k$$

$$a_4 = -\frac{1}{3}\,a_2 = \frac{-a_0}{3}$$

$$+ \cancel{3}\ \frac{1}{\ }\,a_2 = \frac{1}{5}a_0$$

$$(t) = a_1 t + a_0 \left[1 + t^2 - \frac{t^4}{3} + \frac{t^6}{5} - \frac{t^8}{7} \cdots \right]$$

$$= \boxed{a_1 \cancel{(t)} + a_0 \left[1 + t \operatorname{arctg}(t) \right]}$$

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2) $x^2 y'' + x(1-x) y' - 2y = 0$

$\cancel{x}y'' + \frac{1-x}{x} y' - \frac{2}{x^2} y = 0$

$$\boxed{\begin{aligned} p_0 &= 1 \\ q_0 &= -2 \end{aligned}}$$

polinomio indicial

$$p(\lambda) = \lambda^2 + (p_0 - 1)\lambda + q_0$$

$$= \lambda^2 - 2$$

$$\boxed{\lambda_1 = \sqrt{2} \qquad \lambda_2 = -\sqrt{2}}$$

$$y = y_1(x) = x^{\sqrt{2}} \sum_{k=0}^{\infty} a_k x^k = \sum_{k=0}^{\infty} a_k x^{(k+\sqrt{2})}$$

$$y'(x) = \sum_{k=0}^{\infty} (k+\sqrt{2}) a_k x^{k-1+\sqrt{2}}$$

$$y''(x) = \sum_{k=0}^{\infty} (k+\sqrt{2})(k-1+\sqrt{2}) a_k x^{k-2+\sqrt{2}}$$

$$\sum_{k=0}^{\infty} (k+\sqrt{2})(k-1+\sqrt{2}) a_k x^{k+\sqrt{2}} + \sum_{k=0}^{\infty} (k+\sqrt{2}) a_k x^{k+\sqrt{2}} +$$

$$- \sum_{k=0}^{\infty} (k+\sqrt{2}) a_k x^{k+1+\sqrt{2}} + (-2) \cdot \sum_{k=0}^{\infty} a_k x^{k+\sqrt{2}}$$

$$\sum_{k=1}^{\infty} x^{k+\sqrt{2}} \Big[\underbrace{(k+\sqrt{2})(k-1+\sqrt{2})\, a_k + (k+\sqrt{2})\, a_k - 2\, a_k} - (k-1+\sqrt{2})\, a_{k-1} \Big]$$

$$+ a_0 \Big(\underbrace{\sqrt{2}\cdot(\sqrt{2}-1) + \sqrt{2} - 2}_{=0\ \checkmark} \Big) \cdot x^{\sqrt{2}}$$

$$\sum_{k=1}^{\infty} x^{k+\sqrt{2}} \Big[\underbrace{a_k \big((k+\sqrt{2})(k+\sqrt{2}) - 2 \big) - (k-1+\sqrt{2})\, a_{k-1}}_{=0} \Big]$$

$$\boxed{a_k = \frac{k-1+\sqrt{2}}{(k+\sqrt{2})(k+\sqrt{2})-2}\, a_{k-1}}$$

$a_0 :=$

$$a_1 = \frac{\sqrt{2}}{(1+\sqrt{2})^2-2} =$$

$$a_2 = \frac{\sqrt{2}}{(1+\sqrt{2})^2-2} \cdot \frac{(1+\sqrt{2})}{(2+\sqrt{2})^2-2}$$

$$a_3 = \frac{\sqrt{2}}{(1+\sqrt{2})^2-2} \, \frac{(1+\sqrt{2})}{(2+\sqrt{2})^2-2} \, \frac{(2+\sqrt{2})}{(3+\sqrt{2})^2-2}$$

$\therefore$ mismo cuento.

$$y_2(x) = x^{-\sqrt{2}} \sum_{k=0}^{\infty} b_k x^k = \sum_{k=0}^{\infty} b_k x^{k-\sqrt{2}}$$

$\therefore$

$$\sum_{k=0}^{\infty} (k-\sqrt{2})(k-1-\sqrt{2})\, b_k x^{k-\sqrt{2}} + \sum_{k=0}^{\infty} (k-\sqrt{2})\, b_k x^{k-\sqrt{2}}$$

$$- \sum_{k=0}^{\infty} (k-\sqrt{2})\, b_k x^{k+1-\sqrt{2}} - 2 \sum_{k=0}^{\infty} b_k x^{k-\sqrt{2}} = 0$$

$$\Rightarrow b_0 \Big(\underbrace{(-\sqrt{2})\cdot(-1-\sqrt{2}) + (-\sqrt{2}) - 2}_{=0\ \checkmark}\Big) x^{-\sqrt{2}} + \sum_{k=1}^{\infty} \Big\{ b_k \big[(k-\sqrt{2})(k-1-\sqrt{2}) + (k-\sqrt{2}) - 2\big] - b_{k-1}(k-1-\sqrt{2}) \Big\} x^k$$

$$\Rightarrow \boxed{b_k = \frac{(k-1-\sqrt{2})}{(k-\sqrt{2})^2 - 2}\, b_{k-1}}$$

$b_0 := 1$

$$b_1 = \frac{-\sqrt{2}}{(1-\sqrt{2})^2 - 2} = \cancel{1-2\sqrt{2}+2}$$

$$b_2 = \frac{-\sqrt{2}}{(1-\sqrt{2})^2 - 2} \cdot \frac{(1-\sqrt{2})}{(2-\sqrt{2})^2 - 2}$$

$$b_3 = \frac{-\sqrt{2}}{(1-\sqrt{2})^2 - 2} \cdot \frac{1-\sqrt{2}}{(2-\sqrt{2})^2 - 2} \cdot \frac{2-\sqrt{2}}{(3-\sqrt{2})^2 - 2}$$

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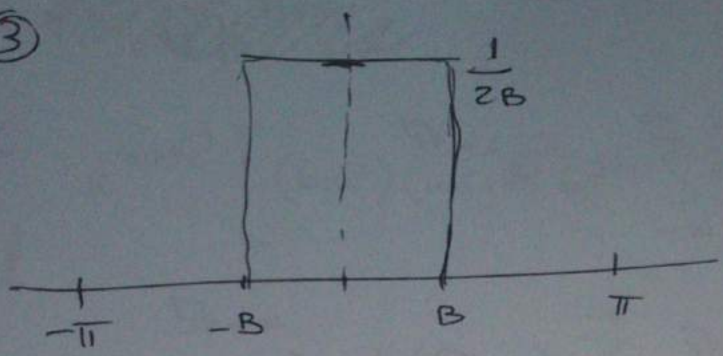
③



②

$$\int_{-\pi}^{\pi} \operatorname{sen}(x)^2\,dx =$$

$$\int_{-\pi}^{\pi} \frac{1-\cos(2x)}{2}\,dx = \frac{1}{\pi}$$

$$\int_{-\pi}^{\pi} f(x)\ \operatorname{sen}(kx)\,dx = 0 \qquad \text{pues } f \text{ es par}$$

$$\int_{-\pi}^{\pi} f(x)\ \cos(kx)\,dx = \int_{-B}^{B} \frac{1}{2B}\cos(kx)\,dx =$$

$$= \frac{1}{2B}\left[\left.\frac{\operatorname{sen}(kx)}{k}\right|_{x=-B}^{x=B} \right]$$

$$= \frac{1}{2Bk}\left[\operatorname{sen}(kB) - \operatorname{sen}(-Bk) \right]$$

$$= \frac{1}{\cancel{2}Bk}\left[\cancel{2}\operatorname{sen}(kB) \right]$$

$$\boxed{Sf(x) = \frac{1}{2\pi} + \frac{1}{\pi}\sum_{k=1}^{\infty} \frac{\operatorname{sen}(kB)}{kB}\cos(kx)}$$

Verificar que $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$

$$\begin{cases} u'' + u' + \lambda u = 0 & 0 < x < \pi \\ u(0) = u(\pi) = 0 \end{cases}$$

$$p(r) = r^2 + r + \lambda$$

$$\boxed{r_{1,2} = \frac{-1 \pm \sqrt{1-4\lambda}}{2}}$$

$u(x) =$

la solución general de la EDO es

$$u(x) = c_1\, e^{\frac{-1+\sqrt{1-4\lambda}}{2}x} + c_2\, e^{\frac{-1-\sqrt{1-4\lambda}}{2}x}$$

Busquemos las constantes.

$$u(0) = 0 \Rightarrow c_1 + c_2 = 0 \quad ; \quad \underline{c_2 = -c_1}$$

$$u(\pi) = 0 \Rightarrow c_1 e^{\frac{-1+\sqrt{1-4\lambda}}{2}\pi} - c_1\, e^{\frac{-1-\sqrt{1-4\lambda}}{2}\pi} = 0$$

$\therefore$

$$c_1\, e^{-\frac{1}{2}\pi} e^{\frac{\sqrt{1-4\lambda}}{2}\pi} - c_1 e^{-\frac{1}{2}\pi} e^{-\frac{\sqrt{1-4\lambda}}{2}\pi} = c_1 e^{-\frac{1}{2}\pi}\left[e^{\frac{\sqrt{1-4\lambda}}{2}\pi} - e^{-\frac{\sqrt{1-4\lambda}}{2}\pi}\right.$$

$$= c_1 e^{-\frac{1}{2}\pi} e^{\frac{-\sqrt{1-4\lambda}}{2}\pi}\left[e^{\sqrt{1-4\lambda}\,\pi} - 1\right] = 0$$

$$\Rightarrow \quad \sqrt{1-4\lambda}\,\cancel{\pi} = 2k\cancel{\pi}\,i$$

$$1-4\lambda = -4k^2$$

$$1+4k^2 = 4\lambda$$

$$\boxed{\frac{1+4k^2}{4} = \lambda_k}$$ autovalores.

autofunciones:

$$u_k(x) = c_1\left[e^{\frac{-1+\sqrt{1-4\lambda_k}}{2}x} - e^{\frac{-1-\sqrt{1-4\lambda_k}}{2}x}\right]$$

$$= c_1\left[e^{\left(-\frac{1}{2}+ki\right)x} - e^{\left(-\frac{1}{2}-ki\right)x}\right]$$

$$= c_1\, e^{-\frac{1}{2}x}\left[e^{kxi} - e^{-kix}\right]$$

$$= c_1\, e^{-\frac{1}{2}x}\,\frac{\mathrm{sen}(kx)}{2i}$$

$$\boxed{u_k(x) = \tilde{c}_1\, e^{-\frac{x}{2}}\,\mathrm{sen}(kx)}$$

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Verif:

$$u' = \underbrace{e^{-x/2}\cos(kx)\cdot k}\ -\ e^{-x/2}\cdot\mathrm{sen}(kx)\,\frac{1}{2}$$

$$u'' = e^{-x/2}\,\mathrm{sen}(kx)\cdot(-k^2)\ \underbrace{-\ e^{-x/2}\cos(2kx)\cdot k}\ +\ \frac{1}{4}e^{-x/2}\,\mathrm{sen}(2kx)$$

$$u''+u' = e^{-x/2}\,\mathrm{sen}(2kx)\left[-\frac{1}{2}+\frac{1}{4}\cdot -k^2\right] = e^{-x/2}\,\mathrm{sen}(2kx)\left[-\frac{1}{\ }-(k^2\right]\ \checkmark$$

$$u'' + u' + \lambda u = 0$$

$$(e^x u')' + \lambda e^x u = 0$$

peso e^x

$$\boxed{\int_0^\pi u_k(x) \cdot u_j(x) \cdot e^x \, dx = 0 \qquad k \neq j}$$

$$= \boxed{p(x)\, W(u, v)(x) \Big|_{x=0}^{x=\pi}} = e^\pi \left(u'(\pi) v(\pi) - u(\pi) v'(\pi) \right) - \left(u'(0) v(0) - u(0) v'(0) \right)$$

$$= \cancel{(-u'(0))} (-v(0)) - \cancel{(-u(0))(-v'(0))} - \cancel{u'v(0) + u(0)v}$$

$$= 0$$

è autoaggiunto

$$p(\pi)\, W[u,v](\pi) - p(0)\, W[u,v](0) = 0$$

$$\begin{cases} u(0) = -e^\pi u(\pi) \\ u'(0) = -u'(\pi) \end{cases}$$

$$v(0) = -$$

$$\langle Lu, v \rangle = \langle u, Lv \rangle$$