2 1-a) $\sum_{i=1}^{n} \frac{5}{(4i-2)(2i+1)} = \frac{5n}{2(2n+1)}, \forall n \in \mathbb{N}, n \geq 1$

- $P(1)$: $\sum_{i=1}^{1} \frac{5}{(4i-2)(2i+1)} = \frac{5}{(4\cdot 1-2)(2\cdot 1+1)} = \frac{5}{2\cdot 3} = \frac{5}{6}$

por otro lado $\frac{5\cdot 1}{2\cdot(2\cdot 1+1)} = \frac{5}{2\cdot 3} = \frac{5}{6}$ $\Big\}$ $P(1)$ es V.

- H.I.: $\sum_{i=1}^{k} \frac{5}{(4i-2)(2i+1)} = \frac{5\cdot k}{2(2k+1)}, \quad k \geq 1.$

T.I.: $\sum_{i=1}^{k+1}$ " $= \frac{5\cdot(k+1)}{2\cdot(2(k+1)+1)}$ → Lo vemos:

$$\sum_{i=1}^{k+1} \text{''} = \sum_{i=1}^{k} \frac{5}{(4i-2)(2i+1)} + \frac{5}{[4(k+1)-2][2(k+1)+1]}$$

$$\underset{H.I.}{=} \frac{5\cdot k}{2\cdot(2k+1)} + \frac{5}{(4k+2)(2k+3)}$$

$$= \frac{5}{2}\left\{\frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}\right\}$$

$$= \frac{5}{2}\left[\frac{k\cdot(2k+3)+1}{(2k+1)(2k+3)}\right] = \frac{5}{2}\cdot\left[\frac{2k^2+3k+1}{(2k+1)\cdot(2k+3)}\right]$$

Quiero ver si: $\frac{5}{2}\left[\frac{2k^2+3k+1}{(2k+1)(2k+3)}\right] = \frac{5\ (k+1)}{2\cdot(2(k+1)+1)}$, esto

ocurre $\longleftrightarrow$ $2k^2+3k+1 = \frac{(k+1)}{(2k+3)}\cdot(2k+1)\cancel{(2k+3)}$

$\longleftrightarrow$ $2k^2+3k+1 = 2k^2+k+2k+1$, vale.

$\therefore \sum_{i=1}^{k+1} \frac{5}{(4i-2)(2i+1)} = \frac{5(k+1)}{2(2(k+1)+1)}$, como queríamos probar